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JAP-003-0011005 Seat No. ________

B. Sc. (Sem. I) (CBCS) (W.E.F. 2016) Examination

November - 2019

MATH - 01 (A) : Calculus

(Elective)

Faculty Code : 003

Subject Code : 0011005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figure to the right indicate full marks of the question.

1 (A) Answer the following : **4**

(1) State the Rolle's Theorem.

(2) Why Roll's Theorem is not applicable to the function $f(x) = x^2 \; x \in [1, 2]$?

(3) Write the Maclaurian's series expansion of cos x up to 5 terms.

(4) True or False : Roll's Mean Value Theorem is a special case of Lagrange's mean value Theorem.

(B) Attempt any **one** : **2**

(1) Express $f(x) = 2x^3 + 7x^2 + x - 6$ in ascending powers of $(x-2)$.

(2) Write Maclaurin's series expansion of e^{2x} up to first four non-zero terms.

(C) Attempt any **one** : **3**

(1) Show that $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta,$ where $0 < \alpha < \theta < \beta < \frac{\pi}{2}.$

(2) Verify Lagrange's mean value Theorem for $f(x) = x^3$ in $[-2, 2].$

(D) Attempt any **one** : **5**

(1) State and prove Lagrange's Mean Value Theorem.

(2) If $0 < u < v,$ then show that

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$$

and deduce that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$

2 (A) Answer the following : **4**

(1) What is the degree and order of the differential equation $(y''')^2 + \left[(y')^2 - 1\right]^3 = 0?$

(2) Write the general form of fisrst order and first degree linear differential equation.

(3) Define : Homogeneous differential equation.

(4) True of False $\frac{dy}{dx} = \frac{x}{y}$ is not a variable separable differential equation.

(B) Attempt any **one** : **2**

(1) Evaluate : $\lim_{x \to 0} \frac{(1+x)^3 - 1}{x}$.

(2) Solve : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

(C) Attempt any **one** : **3**

(1) Solve : $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$.

(2) Evaluate : $\lim_{x \to 0} \frac{\log\left(1 + x^3\right)}{\sin^3 x}$.

(D) Attempt any **one** : **5**

(1) Evaluate : $\lim_{x \to 0} \frac{5 \sin x - 7 \sin 2x + 3 \sin 3x}{\tan x - x}$

(2) Solve the homogeneous differential equation :

$x(x - y)\,dy + y^2\,dx = 0$.

3 (A) Answer the following : **4**

(1) Define : Clairaut's differential equation.

(2) Cheek whether the differential equation $x\,dy + y\,dx = 0$ is exact or not.

(3) Find integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 2$.

(4) State the condition for the equation $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact.

(B) Attempt any **one** : **2**

(1) Solve : $y = xp - p^2 + \log p$, where $p = \frac{dy}{dx}$

(2) Solve : $\left(y^2 - x^2\right) dx + 2xy\, dy = 0$.

(C) Attempt any **one** : **3**

(1) Solve : $p^2 - 5p + 6 = 0$, where $p = \frac{d}{dx}$.

(2) Solve : $x \frac{dy}{dx} + y \log y = xye^x$.

(D) Attempt any **one** : **5**

(1) Solve : $(\cos x + y \sin x)\, dx = (\cos x)\, dy$, $y(\pi) = 0$.

(2) Solve : $p^2 - 6px + 3y = 0$, where $p = \frac{dy}{dx}$.

4 (A) Answer the following : **4**

(1) If roots of auxiliary equation are 1, 2, 3, then write corresponding C. F.

(2) Find $D^3\left(e^{2x}\right)$, where $D = \frac{d}{dx}$.

(3) Find $\frac{1}{D^2}(\sin 2x)$, where $D = \frac{d}{dx}$.

(4) $\frac{1}{D-2} X =$ __________, where $D = \frac{d}{dx}$.

(B) Attempt any **one** : **2**

(1) Solve : $\left(D^4 - 16\right)y = 0$, where $D = \frac{d}{dx}$.

(2) Solve : $\left(D^3 - D^2 + 4D - 4\right)y = e^x$, where $D = \frac{d}{dx}$.

(C) Attempt any **one** : **3**

(1) Solve : $\left(4D^2 - 4D + 1\right)y = e^{x/2}$, where $D = \frac{d}{dx}$.

(2) In usual notation prove that : $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$ if $f(a) \neq 0$.

(D) Attempt any **one** : **5**

(1) Solve : $\left(D^3 + 3D + 2\right)y = \sin 2x$, where $D = \frac{d}{dx}$.

(2) Prove that :

$$\frac{1}{\varphi\left(D^2\right)}\sin(ax+b) = \begin{cases} \frac{1}{\varphi\left(-a^2\right)}\sin(ax+b) & if\ \varphi\left(-a^2\right) \neq 0, \\ \frac{x}{\varphi'\left(-a^2\right)}\sin(ax+b) & if\ \varphi\left(-a^2\right) = 0,\ \varphi'\left(-a^2\right) \neq 0. \end{cases}$$

5 (A) Answer the following : **4**

(1) Write general form of the Cauchy's homogeneous linear differential equations.

(2) Evaluate : $\frac{1}{D-1}e^{x}$.

(3) Evaluate : $\frac{1}{D+1}x$.

(4) True OR False : Cauchy's homogeneous linear differential equations is a special case of Legendry's homogeneous linear differential equations.

(B) Attempt any **one** : **2**

(1) Solve : $\left(x^3D^3 + x^2D^2 - 2\right)y = 0$, where $D = \frac{d}{dx}$.

(2) Solve : $\left(4x^2D^2 + 16xD + 9\right)y = 0$, where $D = \frac{d}{dx}$.

(C) Attempt any **one** : **3**

(1) Solve : $\left(4x^2D^2 + 1\right)y = 19\cos(\log x)$, where $D = \frac{d}{dx}$.

(2) Solve :

$$\left((3x+2)^2 D^2 + 3(3x+2)D - 36\right)y = 3x^2 + 4x + 1,$$

where $D = \frac{d}{dx}$.

(D) Attempt any **one** : **5**

(1) Solve :

$$\left((1+x)^2 D^2 + (1+x) D + 1\right) y = 2\sin\left(\log(1+x)\right),$$

where $D = \dfrac{d}{dx}$.

(1) Solve : $\left(x^2 D^2 - xD + 2\right) y = 6,\ y(1) = 1, y'(1) = 2,$

where $D = \dfrac{d}{dx}$.
